BIBLIOGRAPHY

1. Liapunov, A. M., Collected Works, Vol. 2. M., Fizmatgiz, 1958.
2. Barbashin, E. A., Introduction to the Theory of Stability. M., "Nauka", 1967.
3. Krasovskii, N. N., Theorems on the stability of motions governed by a system of two equations. PMM Vol. 16, №5, 1952.
4. Krasovskii, N. N., On the stability of solutions of a system of two differential equations. PMM Vol. 17, №6, 1953.
5. Pliss, V. A., A qualitative illustration of integral curves on the whole and the construction of the region of stability of a system of two differential equations with arbitrary accuracy. PMM Vol. 17, №5, 1953.
6. Kobyshev, V. A. and Shakhtarin, B. I., Study of a nonlinear system. Differentsial'nye uravneniia, Vol. 2, №8, 1966.
7. Kapranov, M. V., Pull-in range in the phase automatic frequency control. Radiotekhnika, Vol. 11, №12, 1956.
8. Shakhtarin, B. I., A lemma in the theory of phase automatic frequency control. Radiotekhnika i elektronika, Vol. 13, №9, 1968.

Translated by L. K.

UDC 534

ON WAVE FIELDS AND ACUTE-ANGLED EDGES ON WAVE FRONTS IN AN ANISOTROPIC MEDIUM FROM A POINT SOURCE

PMM Vol. 36, №5, 1972, pp. 927-934
I. O. OSIPOV
(Petrozavodsk)
(Received December 27, 1971)

In addition to the results in [1], wave fields of quasi-longitudinal and quasi-transverse elastic vibrations from a point source of an instantaneous pulse type are studied in an anisotropic medium with four elastic constants. Cases are considered when the wave fronts are convex closed curves and when the inner front consists of piecewise-smooth curves forming acute-angled edges.

The solution characterizing the elastic vibrations of quasi-longitudinal and quasi-transverse type SV waves in an infinite anisotropic medium from a point source of instantaneous pulse type placed at the origin is [1]

$$u = \sum_{k=1}^{2} R\left\{c\int^{\theta_k} \zeta\lambda_k w_k(\zeta)\,d\zeta\right\} \qquad (1)$$

$$v = \sum_{k=1}^{2} R\left\{\int^{\theta_k} (a\zeta^2 + d\lambda_k^2 - 1)\,w_k(\zeta)\,d\zeta\right\}$$

The complex variables θ_k and the quantities λ_k are defined by the following relationsips:

$$1-\theta_k\xi+\lambda_k\eta=0 \qquad (\xi=x/t,\ \eta=y/t) \tag{2}$$

$$\lambda_k=\left(\frac{[(b+d)-L\theta_k^2]+(-1)^k\sqrt{Q(\theta_k)}}{2bd}\right)^{1/2} \qquad (k=1,2) \tag{3}$$

$$Q(\theta_k)=[(b+d)-L\theta_k^2]^2-4bd\,(1-a\theta_k^2)(1-d\theta_k^2)$$

$$L=ab+d^2-c^2$$

The functions λ_k are branches of an algebraic function λ which is single-valued on a Riemann surface [2]. The functions w_k are branches of an arbitrary analytic function w which is single-valued on a two-sheeted Riemann surface. The function w must be chosen so that the real parts w_k would vanish on the edges of the slits of planes of the Riemann surface [2, 3], where the functions λ_k take on real values. Wave fronts which are expressed as envelopes of the lines (2) for real values of θ_k and λ_k

$$\xi_k=-\lambda_k'/(\lambda_k-\theta_k\lambda_k'), \qquad \eta_k=-1/(\lambda_k-\theta_k\lambda_k') \tag{4}$$

correspond to the edges of these slits in the $\xi\eta$-plane. For real media of the considered class of anisotropy, the ratios between the elastic constants and the density

$$a=C_{11}/\rho,\ b=C_{22}/\rho,\ d=C_{66}/\rho,\ c=(C_{66}+C_{12})/\rho \tag{5}$$

satisfy the condition

$$a>d,\ b>d,\ d>0,\ K_1=ab-(c-d)>0 \tag{6}$$

Let us limit ourselves below to the consideration of cases when the quantities (5) satisfy the additional condition

$$K_2=ab-(c+d)<0 \tag{7}$$

Under the condition (7) two branch points are real for the inner radical in (3) and two are imaginary [2]

$$\theta_i^\circ=\pm\left(\frac{M\pm\sqrt{-4bdc^2N_1}}{K_1K_2}\right)^{1/2} \tag{8}$$

$$M=(b+d)N_1-(b-d)(a-b)d$$

$$N_1=(a-d)(b-d)-c^2$$

The picture of elastic wave propagation in media satisfying condition (7) depends on the signs of the quantities

$$N_2=(a-d)b-c^2,\ N_3=(b-d)a-c^2 \tag{9}$$

Let us first examine the case when $N_2>0$, $N_3>0$. In this case the Riemann surface

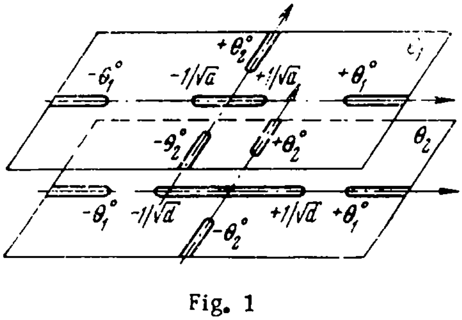

Fig. 1

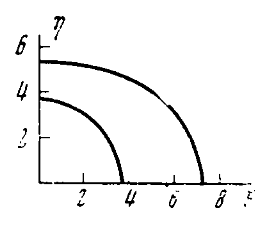

Fig. 2

[2] is represented in Fig. 1; the edge of the slits (θ_i°,∞) of the θ_1- and θ_2-planes are connected crosswise. Let us fix the functions λ_k on the θ_k-planes so that they would be

positive for $\theta_k = i\,\beta$, where β is a sufficiently small positive quantity. The correspondence between points of the $\xi\,\eta$-plane and points of the θ_k-planes is expressed by the relations (2). Substituting the values (3) into the relations (2) and eliminating the radicals, we arrive at the identical equation

$$\begin{gathered}(bd\xi^4 + ad\eta^4 + L\xi^2\eta^2)\,\theta^4 - 2\xi\,(2bd\xi^2 + L\eta^2)\,\theta^3 - \\ [(a+d)\,\eta^4 + (b+d)\,\xi^2\eta^2 - 6bd\xi^2 - L\eta^2]\,\theta^2 + 2\xi\,[(b+d)\,\eta^2 - \\ 2bd]\,\theta + [\eta^4 - (b+d)\,\eta^2 + bd] = 0\end{gathered} \tag{10}$$

Equation (10) has four roots at each point of the $\xi\,\eta$-plane; the roots are identical at points symmetric with respect to the ξ-axis. The complex roots are pairwise conjugate. In the case under consideration, the fronts of the quasi-longitudinal and quasi-transverse waves expressed by (4), are convex, closed curves [3]. Pictured in Fig. 2 are the wave fronts for aragonite [4]

$$C_{11} = 160,\ C_{22} = 86.7,\ C_{66} = 42.7,\ C_{12} = 37.3\ [10^{10}\ \text{dyn/cm}^2],\ \rho = 2.95\ \text{g/cm}^3$$

(the picture is symmetric relative to the ξ- and η-axes). The edges of the slits $(-1/\sqrt{a},\ +1/\sqrt{a})$ of the θ_1-plane and $(-1/\sqrt{d},\ +1/\sqrt{d})$ of the θ_2-plane correspond to the quasi-longitudinal and quasi-transverse wave fronts. All four roots of (10) are real at points of the $\xi\eta$-plane exterior to the quasi-longitudinal wave front. Two of them retain constant values along the tangents to the quasi-longitudinal wave front and belong to the edges of the slit $(-1/\sqrt{a},\ +1/\sqrt{a})$ of the θ_1-plane, and the other two retain constant values along the tangents to the quasi-transverse wave front and belong to the edges of the slit $(-1/\sqrt{d},\ +1/\sqrt{d})$ of the θ_2-plane. In the domain exterior to the quasi-longitudinal wave front, and at points of the front itself, the solution (1) vanishes. Equation (10) has two real and two complex roots at points of the domain included between the wave fronts. The real roots remain constant along the tangents to the quasi-transverse wave front and belong to the edges of the slit $(-1/\sqrt{d},\ +1/\sqrt{d})$ of the θ_2-plane on which terms of the solutions (1) corresponding to $k = 2$ vanish. All four roots of (10) are complex at points of the domain interior relative to the quasi-transverse wave front, and both members in the solution (1) are not zero. The above information about the roots of (10) at points of the domains bounded by the wave fronts does not refer to the roots on sections of the ξ-axis where they have real values. All roots of (10) become infinite at the point $\xi = 0,\ \eta = 0$; the neighborhoods of the origin in the $\xi\,\eta$-plane correspond to the neighborhoods of the infinitely remote points on the θ_1- and θ_2-planes of the Riemann surface.

Let us study the correspondence between points of the wave fields and points of the Riemann surface expressed by (2). Let $\theta_k = \delta_k + i\,\varepsilon_k$ and $\lambda_k = E_k + iF_k$. Then the correspondence between points of the Riemann surface and of the wave fields is expressed by the formulas

$$\xi = -F_k/(\varepsilon_k E_k - \delta_k F_k), \qquad \eta = -\varepsilon_k/(\varepsilon_k E_k - \delta_k F_k) \tag{11}$$

Let us provisionally consider that the quasi-longitudinal wave is propagated on the $\xi_1\eta_1$-plane, and the quasi-transverse wave on the $\xi_2\eta_2$-plane. The subscripts at the coordinate points in (2) and (11), which show to which planes $\xi_k\eta_k$ the points belong, will not yet be disclosed. According to (11), the segments $(\pm 1/\sqrt{a},\ \pm\infty)$ and $(\pm 1/\sqrt{d},\ \pm\infty)$ on the real axes of the θ_1- and θ_2-planes, set in correspondence by the expressions $\theta_k = 1/\xi_k$, correspond to the segments $(-\sqrt{a},\ +\sqrt{a})$ and $(-\sqrt{d},\ +\sqrt{d})$ cut off by the wave fronts on the ξ_1- and ξ_2-axes. The members of the solution (1) corresponding to these segments are different from zero. Parts of the wave fields in the lower $\xi_k\eta_k$

half-plane correspond to the upper θ_k half-planes.

The functions (3) take on the positive real values

$$\lambda_k(i\varepsilon_k) = \left(\frac{[(b+d)+L\varepsilon_k^2]+(-1)^k\sqrt{Q(i\varepsilon_k)}}{2bd}\right)^{1/2} \qquad (12)$$

$$Q(i\varepsilon_k) = [(b+d)+L\varepsilon_k^2]^2 - 4bd(1+a\varepsilon_k^2)(1+d\varepsilon_k^2)$$

on sections $(0, \theta^\circ_2)$ of the positive imaginary semi-axes of the θ_k-planes, i.e. for $\theta_k = i\varepsilon_k$. The first derivatives with respect to the variable ε_k are

$$\lambda_k' = \frac{\varepsilon_k \Psi_k}{2bd\lambda_k(i\varepsilon_k)}, \qquad \Psi_k = L + (-1)^k (K_1K_2\varepsilon_k^2 + M) / \sqrt{Q(i\varepsilon_k)}$$

The conditions $\Psi_1' > 0$ and $\Psi_2' < 0$ are satisfied on the sections $0 \leqslant \varepsilon_k \leqslant \varepsilon^\circ$, where $\varepsilon^\circ = \theta_2^\circ / i$; on the boundaries of the sections

$$\Psi_1(0) = 2d[(b-d)d + c^2]/(b-d), \quad \Psi_1(\varepsilon^\circ) = \infty$$

$$\Psi_2(0) = 2bN_2/(b-d), \quad \Psi_2(\varepsilon^\circ) = -\infty$$

Therefore, the functions Ψ_1 and λ_1' here have positive values, and the functions Ψ_2 and λ_2' change sign from plus to minus at the point

$$\varepsilon_2^* = [-(\sqrt{ad}M + L\sqrt{c^2[c^2-(a-d)(b-d)]}/\sqrt{ad}K_1K_2]^{1/2}$$

Therefore, the function λ_1 increases monotonously on the section $(0, \theta_2^0)$ of the positive imaginary semi-axis of the θ_1-plane; the function λ_2 has a maximum at the point $\theta_2^* = i\varepsilon_2^*$ within the same section on the θ_2-plane, i.e. grows monotonously in the interval $(0, \theta_2^*)$, decreases monotonously in the interval $(\theta_2^*, \theta_2^\circ)$, where $\lambda_1(\theta_2^\circ) = \lambda_2(\theta_2^\circ)$.

It follows from (12) that the sections $-\sqrt{b} \leqslant \eta_1 \leqslant +\eta_2^\circ$ and $-\sqrt{d} \leqslant \eta_2 \leqslant +\eta_2^*$ of the negative η_k semi-axes, set in correspondence by the expressions $\eta_k = -\varepsilon_k / \lambda_k$, where $\eta_2^* < \eta^*_2 < 0$, will correspond to sections of the positive imaginary semi-axes $0 \leqslant \theta_1 \leqslant \theta_2^\circ$ and $0 \leqslant \theta_2 \leqslant \theta_2^*$. The section $+\eta_2^\circ \leqslant \eta_1 \leqslant +\eta_2^*$ on the $\xi_1\eta_1$-plane set in correspondence by the expression $\eta_1 = -\varepsilon_2/\lambda_2$ corresponds to the section $\theta_2^\circ \geqslant \theta_2 \geqslant \theta_2^*$ in the θ_2-plane; in the opposite case there will not be a unique correspondence between point of the Riemann surface and the wave fields. The functions (12) take on the complex values

$$\lambda_k = E \pm Fi \qquad (13)$$

$$E = \sqrt{S+T}/2\sqrt{bd}, \qquad F = \sqrt{S-T}/2\sqrt{bd}$$

$$S = 2\sqrt{bd(1+a\varepsilon_k^2)(1+d\varepsilon_k^2)}, \qquad T = (b+d) + L\varepsilon_k^2$$

on the edges of the slits $(\theta_2^\circ, i\infty)$ of the θ_k-planes.

It follows from (11) that the points of the $\xi_1\eta_1$-plane

$$\xi_1 = \mp F/\varepsilon_k E, \quad \eta_1 = -1/F \qquad (14)$$

correspond to points of the edges of the slits $(\theta_2^\circ, i\infty)$ of the θ_k-planes. The upper (lower, respectively) signs in (13) and (14) correspond to connecting the left (right) edge of the slit in the θ_1-plane to the right (left) edge of the slit in the θ_2-plane. A line in the third quadrant of the $\xi_1\eta_1$-plane correspond to the first connection of the slit edges, and in the fourth quadrant to the second. The ends of these lines coincide at the points $\eta_1 = \eta_2^\circ$ and $\eta_1 = 0$ of the axis of ordinates forming the closed contour P_1 limiting the domain B_1 symmetric relative to this axis within the quasi-longitudinal wave field.

Let A_1 denote the remaining part of the quasi-longitudinal wave field in the lower $\xi_1\eta_1$ half-plane bounded by the wave front and the contour P_1. The upper θ_1 half-plane, set in correspondence by the relationship (2) for $k = 1$, corresponds to the domain A_1 Shifts of the quasi-longitudinal wave field in the domain A_1 are expressed by members of the solution (1) determined on the upper θ_1 half-plane of the Riemann surface. Some domain just in the θ_2 -plane can correspond to the domain B_1. It follows from (11) that complex points of the θ_2-plane satisfying the condition $F_2 = 0$ can correspond to the sections $(\eta_2^*, 0)$ of the η_k-axes, from which we have

$$\delta_2 = \pm [(A + \sqrt{A^2 - B}) / \sqrt{ad} K_1 K_2]^{1/2} \tag{15}$$
$$A = \sqrt{ad}\,[M - (L^2 + 4abd^2)\,\varepsilon_2^2]$$
$$B = K_1 K_2 \{adK_1K_2\varepsilon_2^4 + 2adM\varepsilon_2^2 + N_3 [(b - d)\,d + c^2]\}$$

Only for real values of ε_2 in the section $(\varepsilon_2^*, \infty)$ do (15) define real values of δ_2 belonging to the sections $(0, \pm\infty)$. Therefore, lines L_2 in the first and second quadrants of the θ_2 -plane, which emerge from the point $\theta_2^* = i\varepsilon_2^*$ toward infinity, will correspond to the sections $(\eta_2^*, 0)$ of the negative η_k semi-axes. The lines L_2 bound the domain D_2 which is symmetric relative to the imaginary axis; we denote the rest of the upper θ_2 half-plane by C_2. The domain D_2 in the upper θ_2 half-plane, set in correspondence by the relation

$$1 - \theta_2\,\xi_1 + \lambda_2\,\eta_1 = 0 \tag{16}$$

corresponds to the domain B_1 of the quasi-longitudinal wave field in the lower $\xi_1\eta_1$ half-plane. Shifts of the quasi-longitudinal wave field in the domain B_1 are expressed by members of the solution (1) with $k = 2$, defined in the domain D_2 of the upper θ_2 half-plane. The domain C_2 of the upper θ_2 half-plane, set in correspondence by the relationship (2) with $k = 2$, corresponds to the domain of the quasi-transverse wave field on the lower $\xi_2\eta_2$ half-plane. Shifts of the quasi-transverse wave field in this domain are expressed by members of the solution (1) for $k = 2$, defined in the domain C_2 of the upper θ_2 half-plane.

Pictured in Fig. 3 are grids in the upper θ_k half-planes which correspond to grids of polar coordinates on the wave fields in the lower $\xi_k\eta_k$ half-planes for aragonite (the pictures are symmetrical relative to the imaginary or the ordinate axes).

Now, let us examine the case when $N_2 < 0$ and $N_3 < 0$. Here, the Riemann surface has the form [3] pictures in Fig. 4; the edges of the slits (θ_i°, ∞) of the θ_k-planes are connected crosswise. The external wave front is a convex closed curve and is expressed by (4) on the edges of the slit $(-1/\sqrt{a}, +1/\sqrt{a})$ of the θ_1-plane. The internal wave front consists of piecewise-smooth curves forming acute-angled edges and is expressed by (4) on the edges of the slits $(-\theta_1^\circ, +\theta_1^\circ)$ of the θ_2-plane and $(\pm 1/\sqrt{d}, \pm\theta_1^\circ)$ of the θ_1-plane.

Pictured in Fig. 5 are wave fronts for magnesium sulfate heptahydrate [4]

$$C_{11} = 69.8,\ C_{22} = 52.9,\ C_{66} = 22.2,\ C_{12} = 39,\ \rho = 1.7\ \text{g/cm}^3$$

(the picture is symmetric relative to the ξ- and η-axes).

The piecewise-smooth curves forming the interior wave front are connected at cusps of the first kind located symmetrically relative to the coordinate axes. Sections of the front connecting the cusps in opposite quadrants are convex curves intersecting at points on the coordinate axes. Sections of the front connecting the cusps in adjoining quadrants are concave curves intersecting the coordinate axes at right angles. The interior wave front forms five domains, one of which is central and bounded by sections of the front connecting the nodal points; the remaining four domains adjoin the central domain at the nodal points. Only two tangents can be drawn to the interior wave front from each point exterior relative to the front; four tangents can be drawn from each point within the four domains bounded by the sections of the front connecting the cusps and the nodal points. It is impossible to draw a tangent from points within the central domain to the interior point.

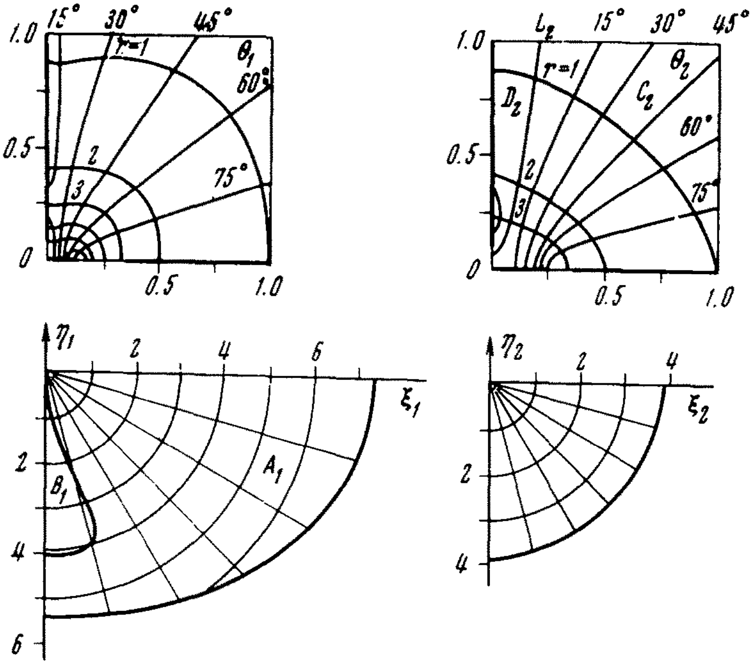

Fig. 3

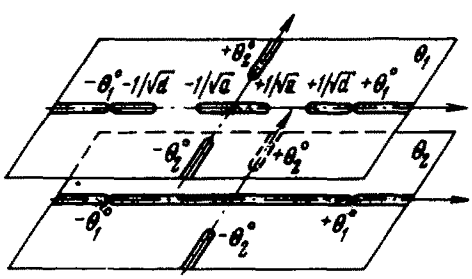

Fig. 4

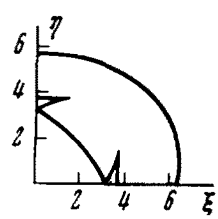

Fig. 5

All four roots of (10) are real at points of the $\xi\eta$-plane exterior to the exterior wave front. Two of them belong to the edges of the slit $(-1/\sqrt{a}, +1/\sqrt{a})$ of the θ_1-plane corresponding to the outer front; and the other two to the edges of the slits $(-\theta_1^\circ, +\theta_1^\circ)$

of the θ_2-plane and $(\pm 1/\sqrt{d}, \pm\theta_1°)$ of the θ_1-plane corresponding to the inner wave front. The solution (1) vanishes at these points.

Equation (10) has two real and two complex roots at points of the domain included between the fronts. The real roots belong to the edges of slits corresponding to the inner front. The members in (1) corresponding to these roots vanish.

All four roots of (10) are real at points of the domains bounded by sections of the inner front connecting the cusps and their nodal points, and they belong to the edges of the slits $(-\theta_1°, +\theta_1°)$ of the θ_2-plane, and $(\pm 1/\sqrt{d}, \pm\theta_1°)$ of the θ_1-plane. The solution (1) vanishes in these domains and there are no elastic vibrations of the kind under consideration.

All four roots of (10) are complex at points of the central domain bounded by sections of the inner front included between the nodal points. The solution (1) in this domain corresponds to quasi-longitudinal and quasi-transverse elastic vibrations.

Therefore, the field of quasi-longitudinal disturbances is a quintuply-connected domain bounded by the outer front and by sections of the inner front connecting the cusps and the nodal points. These sections of the inner front form inner fronts of quasi-longitudinal waves bounding four strips within the quasi-longitudinal wave field wherein there are no quasi-longitudinal and quasi-transverse types of SV disturbances.

A domain bounded by sections of the inner front included between the nodal points is after the quasi-transverse disturbances of SV type. These domains form fronts of the quasi-transverse wave.

Let us study the correspondence between points of the Riemann surface and points of the quasi-longitudinal and quasi-transverse wave fields on the $\xi_1\eta_1$ and $\xi_2\eta_2$-planes.

The function λ_1 takes on imaginary values on the sections $(\pm 1/\sqrt{a}, \pm 1/\sqrt{d})$ of the θ_1-plane, and real values on the edges of the slits $(\pm 1/\sqrt{d}, \pm\theta_1°)$ The functions λ_k have complex values on the edges of the slits $(\pm\theta_1°, \pm\infty)$.

According to (13), the sections $(\pm\sqrt{a}, \pm\sqrt{d})$ and $(\pm\xi_1°, 0)$ of the ξ_1-axis set in correspondence by the expression $\xi_1 = 1/\theta_1$ correspond to the sections $(\pm 1/\sqrt{a}, \pm 1/\sqrt{d})$ and the edges of the slits $(\pm\theta_1°, \pm\infty)$ on the θ_1-plane. Inner quasi-longitudinal wave fronts forming strips containing the sections $(\pm\sqrt{d}, \pm\xi_1°)$ of the ξ_1-axis and the tangents thereto correspond to the edges of the slits $(\pm 1/\sqrt{d}, \pm\theta_1°)$ of the θ_1-plane. The sections $(\pm\xi_1°, 0)$ of the ξ_2-axis, set in correspondence by the expression $\xi_2 = 1/\theta_2$, correspond to the slits $(\pm\theta_1°, \pm\infty)$ of the θ_2-plane.

According to (12), the function λ_1 is a positive monotonously increasing real function on the section $(0, \theta_2°)$ of the positive imaginary semi-axis of the θ_1-plane in the case under consideration; the function λ_2 is a positive monotonously decreasing real function on the same section of the θ_2-plane, hence, $\lambda_1(\theta_2°) = \lambda_2(\theta_2°)$.

It follows from (11) that the section $-\sqrt{b} \leqslant \eta_1 \leqslant \eta_2°$ of the negative η_1 semi-axis set in correspondence by the expression $\eta_1 = -\varepsilon_1/\lambda_1$, corresponds to the section $0 \leqslant \theta_1 \leqslant \theta_2°$ of the positive imaginary semi-axis of the θ_1-plane. The section $\eta_2° \leqslant \eta_1 \leqslant -\sqrt{d}$ of the negative η_1 semi-axis set in correspondence by the expression $\eta_2 = -\varepsilon_2/\lambda_2$, corresponds to the section $\theta_2° \geqslant \theta_2 \geqslant 0$ of the positive imaginary semi-axis of the θ_2-plane, in the opposite case there will be no one-to-one correspondence between points of the Riemann surface and the wave fields.

The functions λ_k take on complex values represented by (13) on the edges of the slits $(\theta_2°, i\infty)$ of the θ_k-planes. A line in the third, (fourth, respectively) quadrant of the quasi-longitudinal wave field corresponds to connecting the left (right) edge of the slit in the θ_1-plane to the right (left) edge of the slit in the θ_2-plane. The ends of these lines coincide at the points $\eta_1 = \eta_2^\circ$ and $\eta_1 = 0$ of the ordinate axis to form a closed contour P_1 expressed by the functions (14).

The domain of the quasi-longitudinal wave field with the external side of the closed contour P_1 in the lower $\xi_1\eta_1$ half-plane is denoted by A_1. The upper θ_1 half-plane set in correspondence by the relationship (2) at $k = 1$ corresponds to the domain A_1. The shifts in this domain are expressed by members of the solution (1) defined on the upper θ_1 half-plane.

The strip of the quasi-longitudinal wave field containing the section $(-\sqrt{d}, \eta_2^*)$ of the negative η_1 half-axis is within the domain bounded by the contour P_1. According to [3], some section $(-\theta_2^*, +\theta_2^*)$ on the upper edge of the slit $(-\theta_1^\circ, +\theta_1^\circ)$ of the θ_2-plane corresponds to the boundary of this strip and the tangent thereto. The domain of the quasi-longitudinal wave field included between the contour P_1 and the strip boundaries is denoted by B_1.

According to (4), the points $\pm\theta_2^*$ corresponding to the points η_2^* on the η_1- and η_2-axes satisfy the equation $\lambda_2' = 0$ and are determined by the expression

$$\theta_2^* = \left(\frac{\sqrt{ad}M + \sqrt{adM^2 - K_1K_2N_3[(b-d)d + c^2]}}{\sqrt{ad}K_1K_2}\right)^{1/2} \tag{17}$$

In the case under consideration, for real values of ε_2 in the interval $(0, \infty)$ the expression (15) defines real values of δ_2 in the intervals $(\pm\delta_2^*, \pm\infty)$, where $\delta_2^* = \theta_2^*$.

Therefore, lines L_2 going from the points $\pm\theta_2^*$ of the upper edge of the slit $(-\theta_1^\circ, +\theta_1^\circ)$ to infinity in the first and second quadrants of the θ_2-plane correspond to the sections $(\eta_2, 0)$ of the negative η_1 and η_2 semi-axes. Let D_2 denote the domain bounded by the lines L_2 and the section $(-\theta_2^*, -\theta_2^*)$ of the upper edge of the slit $(-\theta_1^\circ, +\theta_1^\circ)$, in the upper θ_2 half-plane, and let C_2 denote the rest of the upper θ_2 half-plane.

The domain D_2 in the upper θ_2 half-plane set in correspondence by (16) corresponds to the domain B_1 of quasi-longitudinal wave field in the lower $\xi_1\eta_1$ half-plane. Shifts of the quasi-longitudinal wave field in this domain are expressed by members of the solution (1) for $k = 2$ defined in the domain D_2 of the upper θ_2 half-plane.

The domain C_2 of the upper θ_2 half-plane set in correspondence by the relationship (2) for $k = 2$ corresponds to the domain of the quasi-transverse wave field in the lower $\xi_2\eta_2$ half-plane. Shifts in this domain are expressed by members of the solution (1) for $k = 2$ defined in the domain C_2 of the upper θ_2 half-plane.

Pictured in Fig. 6 are grids on the upper θ_k half-planes corresponding to grids of polar coordinates on the wave fields in the lower $\xi_k\eta_k$ half-planes for magnesium sulfate heptahydrate (the pictures are symmetrical relative to the imaginary or ordinate axes).

Cases when the values of N_2 and N_3 have opposite sign are the passage from the case just considered to another not substantially different case, and can be analyzed easily.

It is assumed in [1] that the members with $k = 1$ in the solution (1) express quasi-longitudinal disturbances, and with $k = 2$ quasi-transverse disturbances of SV type. The investigations performed herein of the wave fields for media satisfying the condition (7) show that the quasi-longitudinal disturbances cannot be expressed just by single members of the solution (1) defined on the θ_1-plane of the Riemann surface. In a certain domain of the wave field the quasi-longitudinal disturbances are expressed by members of the solution (1) defined on the θ_2-plane. Results of investigations show that the wave picture in anisotropic media has negative singularities depending on the relations of the other constants. If at least one of the values of (9) is less than zero, the inner wave front has acute-angled edges. In these cases the quasi-transverse wave field is bounded by sections of the inner front connecting the nodal points. The sections of the inner front forming the acute-angled edges are inner fronts of quasi-longitudinal waves bounding strips within this wave field in which there are no disturbances expressed by the solution

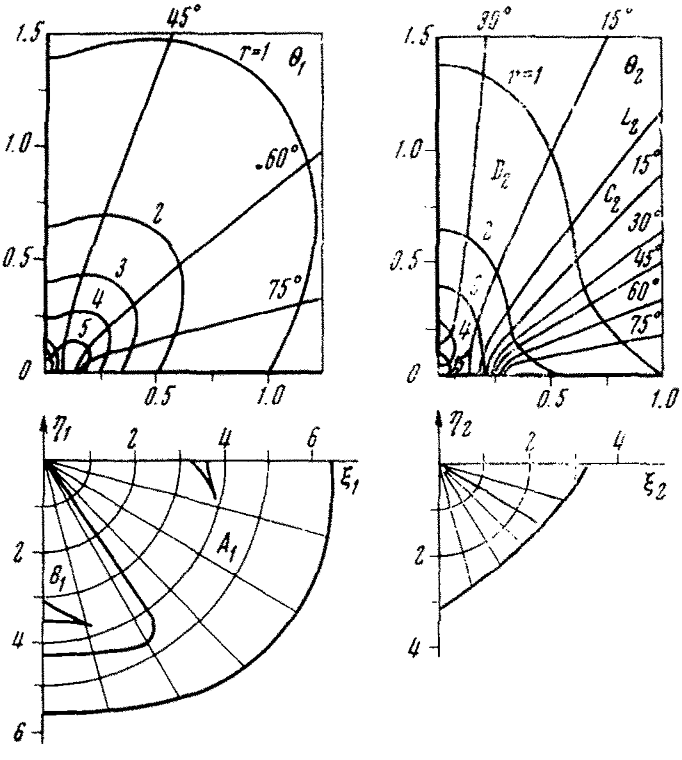

Fig. 6

(1). The quasi-longitudinal disturbance field is a quintuply-connected domain for $N_3 < 0$ and $N_5 < 0$ or a triply-connected domain upon compliance with one of the conditions.

BIBLIOGRAPHY

1. Sveklo, V. A., Elastic vibrations of an anisotropic body. Uch. Zap. Leningrad Univ., Ser. Matem. Nauk, №17, 1949.
2. Osipov, I. O., On the plane problem of a point source of vibrations of the instantaneous pulse type within an anisotropic medium. In: Elastic and Elastic-Plastic Wave Propagation. Tashkent,"FAN," 1969.
3. Osipov, I. O., On the two-dimensional problem of propagation of elastic waves due to a point source in an anisotropic medium. PMM Vol. 33, №3, 1969.
4. Aleksandrov, K. S. and Ryzhova, T. V., Elastic properties of crystals. Kristallografiia, Vol. 6, №2, 1961.

Translated by M. D. F.